The Flow of Water Through an Opening in a Pierced Plate.

BY ROBERT BRIGGS.

(Read before the American Philosophical Society, August 17, 1877.)

At the meeting of the Society on the 3d of November, 1876, I presented an hypothesis of the origin of the form of the *vena contracta* under certain conditions stated in the paper then offered. It was shown that on the assumption that the efflux occurred from the layer or strata of water under greatest pressure of water column, at the maximum velocity due to that column, the least section of the vena contracta would have half the area of the opening of efflux, provided the effect of frictional adhesion of the water to the bottom of the vessel and the effect of the internal friction or viscosity of the water were not considered. And it was noticed that the effect from these causes *tended* to enlarge the least section of the vein and increase the quantity of effluent water.

Referring to the words of the paper : "If however there is admitted to exist a certain

Fig 5
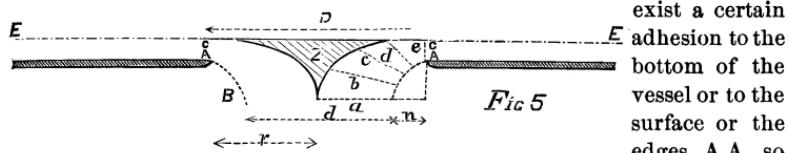
adhesion to the bottom of the vessel or to the surface or the edges A A, so that the velocity of a particle on A B is less than that fully due to the head; the surface (d) would then become larger than $\frac{1}{2}$ D, the dimension C A would be properly increased to give a corresponding area of efflux, and the conoid Z would also have such contour as would permit the uniformity of flow of each and every particle of the liquid at unchanged velocity, in any section of the vena contracta transverse to the flow. This increase of dimension of the cross section d, and the effect of the descending pencil in accelerating the flow through it, can be taken as sufficient to account for Weisbach's observed value of $d = 0.8D$, and the position of the plane of least section will be found at about $\frac{1}{4}$ D below the orifice as has been before quoted."

A further illustration of this subject can be instituted by accepting the observed value of the least section of *vena contracta*, which is found to be 0.64D in place of the hypothetical one of $\frac{1}{2}$ D, and by deducing the form of the effluent vein backwards to the strata of water under greatest pressure. Thus, let it be supposed that Fig. 6 represents (as in Fig. 5)

Fig. 6
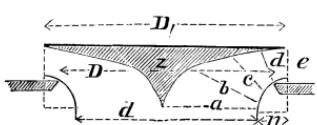
an opening in a thin plate, guarded or protected by a Disc Z, of such contour and so placed that a current flowing towards the opening shall obtain the maximum velocity due to the head, and be diverted from its horizontal to the vertical direction without change of velocity of any particle of the current. The contour of the vena contracta from

the edge of the aperture to the plane of least section is taken to be an arc of a circle—the internal surface of a segment of a ring. Let D be the diameter of the opening in the plate. Suppose d, the diameter of the least section of the *vena contracta,* to have the value given by observation, d = 0.8D. Then following the previous conditions of form of the conoid Z we have, the diameter of the Disc = D_1 = 1.13137D, and the radius of the arc of contour = n = 0.16569D. It will now be observed that the line of the arc of contour, if it is continued within the opening to supposed point of horizontal efflux—the circle of periphery of the disc, gives a strata of water f (shown more distinctly in Fig. 7), which is cut off from the effluent stream. This strata has its greatest thicknesss of f = 0.01358D. These suppositions place the plane of least section = 0.152D below the opening.
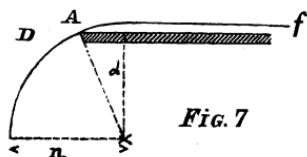
Fig. 7

In Fig. 8 will be seen similar delineation of the contour of the *vena con-tracta,* and the lines of the current of maximum constant velocity, as modified by placing the plane of least section at its observed position, or 0.25D, below the opening in the plate. The contour of the *vena contracta* is here depicted as an arc of an ellipsis which has 0.166D for its minor radius and 0.275D nearly for its major one, which will approximate closely to the true parabolic form

Fig. 8

as suggested in the first paper. The thickness of the film or strata f which represents the resistance arising from friction of water against the bottom and at *the edge* of the aperture now becomes about 0.025D. The angle *a* which the current makes with the edge of the aperture becomes about 35°.
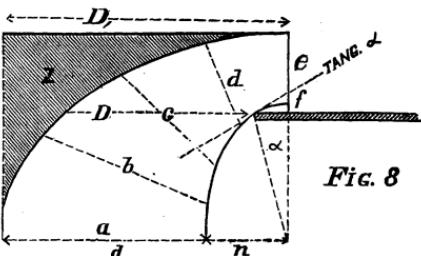
If these suppositions are correct, a re-entering mouth-piece, shaped to conform to the upper part of the elliptical arc would give the same contour and sections to the *vena contracta* as that now found to proceed from free discharge at a plain aperture. It would seem also from the tenor of this discussion that by substituting a re-entering curve at A fig. 7, making the bottom of the vessel to conform to a reversal of the curve A f, giving the *reversed* elliptical arc *a* at the edge of the orifice, so that the tangent of the curvature upwards at the edge should be about 35°, we should then obtain the theoretic least section from a frictionless horizontal surface of = half the area of the opening. And that such a form would be equally effective with the re-entering tube of Mr. Froude, in giving the current at the edge of the aperture its horizontal direction of least resistance accompanied by the greatest liquid pressure.

The Deviating Forces of an Unsymmetrically Balanced Fly-wheel.

Mr. Briggs mentioned that he had not found in the text books of applied or practical Mechanics—Morin, Rankine, Weisbach, Fairbairn or others— any proper consideration had been given to the strains on the axis of a fly-wheel, which, correctly balanced with regard to the gravity of its masses, and also in the plane of rotation, yet without symmetry of position or mass of the balanced parts, is then accelerated or retarded to meet the usual requirements of a regulator of power. The fact that a fly-wheel must be balanced in one plane to run without vibratory effect at any given speed and when thus balanced the centrifugal forces of the parts will be in equilibrium and the axis permanent is fully stated by all recent writers, but the condition of permanency of axis when an unsymmetrically balanced fly-wheel gives out or absorbs force has not been discussed.

The following elementary case shows the proposition distinctly : Let it be supposed that a fly-wheel were formed of a pair of unequal weights at the extremities of arms (radii) of such length as will place the axis in the centre of gravity of the system, thus :

$$\begin{matrix} \text{M} & & \text{m} \\ \text{O}\!\!-\!\!-\!\! & \!\!-\!\!-\!\! & \!\!-\!\!-\text{o} \\ \text{V} & \text{r} \quad \text{R} & \text{V} \end{matrix}$$
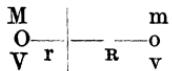
Where M m $=$ the masses and r R $=$ the radii. Let V_1, V_2 and v_1, v_2 represent the two velocities. The admitted energy from the change of velocities of the masses is thus expressed by the equation—

$$F = \left[M (V_1{}^2 - V_2{}^2) + m(v_1{}^2 - v_2{}^2) \right] \div 2g \qquad (1)$$

But from the condition of balancing $m = M\frac{r}{R}$; $v_1 = V_1\frac{R}{r}$; and $v_2 = V_2\frac{R}{r}$

$$\therefore\ F = \left[M (V_1{}^2 - V_2{}^2) + M\frac{r}{R}[V_1{}^2(\frac{R}{r})^2 + V_2{}^2(\frac{R}{r})^2] \right] \div 2g \quad (2)$$

$$F = M\left[(1 + \frac{R}{r}(V_1{}^2 - V_2{}^2)) \right] \div 2g \qquad (3)$$

Showing that the ratio of force given out by the two halves of the fly-wheel under any change of velocity, during any instant of time, will be unity, and the axis be in equilibrium, when $1 = R \div r$ and in no other case, and the masses and velocities become equal in the same case.

This condition of unsymmetrical balancing of fly-wheels is by no means an unusual one. The castings of fly-wheels of steam engines and more especially of pulleys for transmission of force which act generally more or less as fly-wheels, are rarely of such uniformity as not to require balancing, —nearly always done on the rim of the wheel, regardless of point of inequality, which is more frequently in the arms than in the rim.

Perhaps the most striking instance is the case of the vertical blowing engine, where the whole weight of the pistons, crossheads and rods rests upon crank pins inserted in the arms of two fly-wheels at points from one-fourth to one-third the radii of the rim, which weight is counteracted by a suitable load at the rim opposite the crank pins. It is then found that much less load is needed to give comparative steadiness of motion than

what would be required to balance the parts, and that the blowing engine must be balanced to run at a given speed and thus be liable to definite changes of motion of the fly-wheel each stroke. In all steam engines with single cylinders it must be recognized that during an instant of the stroke, the fly-wheel must, solely and unaided, maintain the speed and give out the whole power of the engine *by retardation*, while in most engines, during a considerable portion of the stroke, the fly-wheel is aiding, or assisting to impel, the shaft of transmission ; of course receiving a corresponding impulse from other portions of the same stroke.

The unbalanced forces which result from changes of speed of rotation of these unsymmetrical wheels, are transformed into pressures at the axes and have to be sustained by the bearings and resisted by the frame works which carry or support the same, in addition to any strain, proceeding from the mechanism employed in giving rotation or in transmission of power. As pressure or load upon the bearings, the increment of heat derived from friction may cause the total heat to surpass the limit of dispersion in cases where the direct weight of the fly-wheel, approach, as they frequently do, the maximum load of practical endurance on the bearing surfaces. The apparently unaccountable heating of some fly-wheel bearings, where the absolute pressures from load or work are not so great as to cause heating, has been noticed by all practical mechanics, and the considerations now presented offer a reasonable hypothesis in explanation.

In Mahan's Moseley's Mechanics will be found some mathematical investigations leading in this direction, see appendix notes D and E, but a study of these forces and an application of the theorem to the special case of a fly-wheel regulating force or power is needed to complete the theory of practical mechanical construction.

Description of the Wilcox Spouting Water-Well.

By Chas. A. Ashburner, M. S., Assistant Geological Survey.

(*Read before the American Philosophical Society September* 21, 1877.)

The Wilcox Spouting Water-Well for the last nine months has attracted considerable attention, from the immense columns of water and gas which are periodically (every seven minutes) thrown up into air to a height of from 85 to 115 feet. The well is located in the valley of West Clarion Creek, just north of the southern boundary of McKean County, Pennsylvania, and five miles north of Wilcox, a station on the Philadelphia and Erie Railroad 104 miles east of the City of Erie.

The history of the well may be briefly stated as follows :

The Wilcox Well No. 1, or the old Adams Well, was drilled in 1864 (?)